Available online at www.sciencedirect.com
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J. Math. Anal. Appl. 292 (2004) 423–432
Journal of MATHEMATICAL ANALYSIS AND APPLICATIONS
www.elsevier.com/locate/jmaa

Generalized difference sequence spaces and their dual spaces

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Received 8 July 2003
Submitted by S. Kaijser

Abstract

The definition of the $p\alpha$-, $p\beta$- and $p\gamma$-duals of a sequence space was defined by Et [Internat. J. Math. Math. Sci. 24 (2000) 785–791]. In this paper we compute $p\alpha$- and N-duals of the sequence spaces $\Delta_v^m(X)$ for $X = \ell_\infty$, c and c_0, and compute β- and γ-duals of the sequence spaces $\Delta_v^m(X)$ for $X = \ell_\infty$, c and c_0.

Keywords: Difference sequence spaces; N-dual; $p\alpha$-, $p\beta$- and $p\gamma$-duals

1. Introduction

ω denotes the space of all scalar sequences and any subspace of ω is called a sequence space. Let ℓ_∞, c and c_0 be the linear spaces of bounded, convergent and null sequences $x = (x_k)$ with complex terms, respectively, normed by

$$\|x\|_\infty = \sup_k |x_k|,$$

where $k \in \mathbf{N} = \{1, 2, \ldots\}$, the set of positive integers.

Throughout the paper X will denote one of the sequence spaces ℓ_∞, c or c_0.

The notion of difference sequence spaces was introduced by Kızmaz [2]. It was generalized by Et and Çolak [4] as follows.

Let m be a non-negative integer. Then

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0022-247X/$ – see front matter
doi:10.1016/j.jmaa.2003.12.006

$$\Delta^m(X) = \{x = (x_k)\colon\ (\Delta^m x_k) \in X\},$$

where $\Delta^m x_k = \Delta^{m-1} x_k - \Delta^{m-1} x_{k+1}$ for all $k \in \mathbf{N}$.

Let $v = (v_k)$ be any fixed sequence of non-zero complex numbers. Et and Esi [5] generalized the above sequence spaces to the following sequence spaces:

$$\Delta_v^m(X) = \{x = (x_k)\colon\ (\Delta_v^m x_k) \in X\},$$

where $\Delta_v^m x_k = \Delta_v^{m-1} x_k - \Delta_v^{m-1} x_{k+1}$, $\Delta_v^m x_k = \sum_{i=0}^m (-1)^i \binom{m}{i} v_{k+i} x_{k+i}$ for all $k \in \mathbf{N}$.

The sequence spaces $\Delta_v^m(X)$ are Banach spaces normed by

$$\|x\|_\Delta = \sum_{i=1}^m |v_i x_i| + \|\Delta_v^m x\|_\infty.$$

Now we define

$$\Delta_v^{(m)} x_k = \sum_{i=0}^m (-1)^i \binom{m}{i} v_{k-i} x_{k-i}.$$

Remark. $(\Delta_v^m x_k) \in X$ if and only if $(\Delta_v^{(m)} x_k) \in X$.

Now for $x \in \Delta_v^{(m)}(X)$ define

$$\|x\|_{\Delta'} = \sup_k |\Delta_v^{(m)} x_k|.$$

It can be shown that $(\Delta_v^{(m)}(X), \|\cdot\|_{\Delta'})$ is a BK-space and the norms $\|x\|_\Delta$ and $\|x\|_{\Delta'}$ are equivalent.

Let us define the operator

$$D : \Delta_v^m(X) \to \Delta_v^m(X)$$

by $Dx = (0, 0, \ldots, x_{m+1}, x_{m+2}, \ldots)$, where $x = (x_1, x_2, \ldots, x_m, \ldots)$. It is trivial that D is a bounded linear operator on $\Delta_v^m(X)$. Furthermore the set

$$D[\Delta_v^m(X)] = D\Delta_v^m(X) = \{x = (x_k)\colon\ x \in \Delta_v^m(X),\ x_1 = x_2 = \cdots = x_m = 0\}$$

is a subspace of $\Delta_v^m(X)$ and normed by $\|x\|_\Delta = \|\Delta_v^m x\|_\infty$ in $D\Delta_v^m(X)$. $D\Delta_v^m(X)$ and X are equivalent as topological spaces, since

$$\Delta_v^m : D\Delta_v^m(X) \to X, \quad \text{defined by } \Delta_v^m x = y = (\Delta_v^m x_k), \tag{1}$$

is a linear homeomorphism. Let X' and $[D\Delta_v^m(X)]'$ denote the continuous duals of X and $D\Delta_v^m(X)$, respectively. It can be shown that

$$T : [D\Delta_v^m(X)]' \to X', \quad f_\Delta \to f_\Delta \circ (\Delta_v^m)^{-1} = f,$$

is a linear isometry. So $[D\Delta_v^m(X)]'$ is equivalent to X'.

2. Dual spaces

In this section we give $p\alpha$-, N-, β- and γ-duals of $\Delta_v^m(X)$.

The following results can be found in [1,4].

Let m be a positive integer. Then there exist positive constants M_1 and M_2 such that

$$M_1 k^m \leqslant \binom{m+k}{k} \leqslant M_2 k^m, \quad k = 0, 1, 2, \ldots, \tag{2}$$

$$\sum_{k=0}^{n} \binom{n+m-k-1}{m-1} = \binom{n+m}{m} = \binom{n+m}{n}, \tag{3}$$

$$x \in \Delta_v^m(\ell_\infty) \quad \text{implies} \quad \sup_k k^{-m} |v_k x_k| < \infty. \tag{4}$$

Lemma 2.1 [2]. *Let (p_n) be a sequence of positive numbers increasing monotonically to infinity.*

(i) *If* $\sup_n |\sum_{v=1}^{n} p_v a_v| < \infty$, *then* $\sup_n |p_n \sum_{k=n+1}^{\infty} a_k| < \infty$.
(ii) *If* $\sum_k p_k a_k$ *is convergent, then* $\lim_n p_n \sum_{k=n+1}^{\infty} a_k = 0$.

Definition 2.2 [3]. Let E be a sequence space, $p > 0$ and define

$$E^{p\alpha} = \left\{ a = (a_k): \sum_k |a_k x_k|^p < \infty, \ \forall x \in E \right\},$$

$$E^{p\beta} = \left\{ a = (a_k): \sum_k (a_k x_k)^p \text{ is convergent}, \ \forall x \in E \right\},$$

$$E^{p\gamma} = \left\{ a = (a_k): \sup_n \left| \sum_{k=0}^{n} (a_k x_k)^p \right| < \infty, \ \forall x \in E \right\},$$

$$E^N = \left\{ a = (a_k): \lim_k a_k x_k = 0, \ \forall x \in E \right\}.$$

Then $E^{p\alpha}$, $E^{p\beta}$, $E^{p\gamma}$ and E^N are called $p\alpha$-, $p\beta$-, $p\gamma$- and N-duals of E, respectively. It can be shown that $E^{p\alpha} \subset E^{p\beta} \subset E^{p\gamma}$ and if $E \subset F$, then $F^\eta \subset E^\eta$ for $\eta = p\alpha, \ p\beta, \ p\gamma$ and N. If we take $p = 1$ in this definition, then we obtain the α-, β- and γ-duals of E.

Theorem 2.3. *Let $0 < p < \infty$. Then*

(i) $[\Delta_v^m(\ell_\infty)]^{p\alpha} = [\Delta_v^m(c)]^{p\alpha} = [\Delta_v^m(c_0)]^{p\alpha} = U_1$,
(ii) $U_1^{p\alpha} = U_2$,

where

$$U_1 = \left\{ a = (a_k): \sum_{k=1}^{\infty} k^{pm} \left|v_k^{-1}\right|^p |a_k|^p < \infty \right\},$$

$$U_2 = \Big\{ a = (a_k)\colon \sup_k k^{-pm} |v_k|^p |a_k|^p < \infty \Big\}.$$

Proof. (i) Let $a \in U_1$ and $x \in \Delta_v^m(\ell_\infty)$. Then

$$\begin{aligned}\sum_{k=1}^{\infty} |a_k x_k|^p &= \sum_{k=1}^{\infty} k^{pm} \big|v_k^{-1}\big|^p |a_k|^p k^{-pm} |v_k|^p |x_k|^p \\ &\leqslant \sup_k k^{-pm} |v_k x_k|^p \sum_{k=1}^{\infty} k^{pm} \big|a_k v_k^{-1}\big|^p < \infty.\end{aligned}$$

Hence $a \in [\Delta_v^m(\ell_\infty)]^{p\alpha}$.

Conversely suppose that $a \in [\Delta_v^m(c_0)]^{p\alpha}$ and $a \notin U_1$. Then there exists a strictly increasing sequence (n_i) of positive integers n_i with $n_1 < n_2 < \cdots$, such that

$$\sum_{k=n_i+1}^{n_{i+1}} |v_k|^{-p} k^{pm} |a_k|^p > i^p.$$

Define $x \in \Delta_v^m(c_0)$ by

$$x_k = \begin{cases} 0, & 1 \leqslant k \leqslant n_1, \\ k^m v_k \operatorname{sgn} \frac{a_k}{i}, & n_i < k \leqslant n_{i+1}. \end{cases}$$

Then we have

$$\begin{aligned}\sum_{k=1}^{\infty} |a_k x_k|^p &= \sum_{k=n_1+1}^{n_2} |a_k x_k|^p + \cdots + \sum_{k=n_i+1}^{n_{i+1}} |a_k x_k|^p \\ &= \sum_{k=n_1+1}^{n_2} k^{pm} |v_k|^{-p} |a_k|^p + \cdots + \frac{1}{i^p} \sum_{k=n_i+1}^{n_{i+1}} k^{pm} |v_k|^{-p} |a_k|^p \\ &> 1 + 1 + \cdots = \sum_{i=1}^{\infty} 1 = \infty.\end{aligned}$$

This contradicts to $a \in [\Delta_v^m(c_0)]^{p\alpha}$. Hence $a \in U_1$. This completes the proof of (i).

(ii) Let $a \in U_2$ and $x \in U_1$. Then

$$\begin{aligned}\sum_{k=1}^{\infty} |a_k x_k|^p &= \sum_{k=1}^{\infty} k^{pm} \big|v_k^{-1}\big|^p |a_k|^p k^{-pm} |x_k|^p |v_k|^p \\ &\leqslant \sup_k \big(k^{-pm} |a_k|^p |v_k|^p\big) \sum_{k=1}^{\infty} k^{pm} \big|v_k^{-1}\big|^p |x_k|^p < \infty.\end{aligned}$$

Hence $a \in U_1^{p\alpha}$.

Now suppose that $a \in U_1^{p\alpha}$ and $a \notin U_2$. Then we have

$$\sup_k k^{-pm} |v_k|^p |a_k|^p = \infty.$$

Hence there is a strictly increasing sequence $(k(i))$ of positive integers $k(i)$ such that

$$\left[k(i)\right]^{-pm}|v_{k(i)}|^p|a_{k(i)}|^p > i^m.$$

We define the sequence x by

$$x_k = \begin{cases} |a_{k(i)}|^{-p}, & k = k(i), \\ 0, & k \neq k(i). \end{cases}$$

Then we have

$$\sum_{k=1}^{\infty} k^{pm}\left|v_k^{-1}\right|^p|x_k|^p = \sum_{i=1}^{\infty}\left[k(i)\right]^{pm}|v_{k(i)}|^{-p}|a_{k(i)}|^{-p} \leqslant \sum_{i=1}^{\infty} i^{-m} < \infty, \quad m \geqslant 2.$$

Hence $x \in U_1$ and $\sum_{k=1}^{\infty}|a_k x_k|^p = \sum_{k=1}^{\infty} 1 = \infty$. This contradicts to $a \in U_1^{p\alpha}$. Hence $a \in U_2$. $\square$

If we take $v_k = 1$, for all $k \in \mathbf{N}$ in Theorem 2.3 then we obtain the following corollary.

Corollary 2.4. *Let $0 < p < \infty$. Then we have*

(i) $[\Delta^m(\ell_\infty)]^{p\alpha} = [\Delta^m(c)]^{p\alpha} = [\Delta^m(c_0)]^{p\alpha} = G_1$,
(ii) $G_1^{p\alpha} = G_2$,

where

$$G_1 = \left\{a = (a_k)\colon \sum_{k=1}^{\infty} k^{pm}|a_k|^p < \infty\right\},$$

$$G_2 = \left\{a = (a_k)\colon \sup_k k^{-pm}|a_k|^p < \infty\right\}.$$

Lemma 2.5. *If $x \in \Delta_v^m(c_0)$, then $\binom{m+k}{k}^{-1} v_k x_k \to 0$ $(k \to \infty)$.*

Proof. Proof follows from (2)–(4). $\square$

Theorem 2.6. *Let m be a positive integer. Then*

$$\left[\Delta_v^m(\ell_\infty)\right]^N = \left[\Delta_v^m(c)\right]^N = M_1(v) \quad \textit{and} \quad \left[\Delta_v^m(c_0)\right]^N = M_2(v),$$

where

$$M_1(v) = \left\{a = (a_n)\colon\ v_n^{-1} n^m a_n \to 0,\ n \to \infty\right\}$$

and

$$M_2(v) = \left\{a = (a_n)\colon \sup_n \left|\sum_{k=0}^{n} \binom{n+m-k-1}{m-1} v_n^{-1} a_n\right| < \infty\right\}.$$

Proof. The proof of the part $[\Delta_v^m(\ell_\infty)]^N = [\Delta_v^m(c)]^N = M_1(v)$ is easy. We only show that $[\Delta_v^m(c_0)]^N = M_2(v)$. Let $a \in M_2(v)$ and $x \in \Delta_v^m(c_0)$. Then by Lemma 2.5 and (3) we obtain

$$\begin{aligned}\lim_n a_n x_n &= \lim_n \left(\sum_{k=0}^{n} \binom{n+m-k-1}{m-1}\right) v_n^{-1} a_n \left(\sum_{k=0}^{n} \binom{n+m-k-1}{m-1}\right)^{-1} v_n x_n \\ &= 0.\end{aligned}$$

Hence $a \in [\Delta_v^m(c_0)]^N$.

Now let $a \in [\Delta_v^m(c_0)]^N$. Then $\lim_n a_n x_n = 0$ for all $x \in \Delta_v^m(c_0)$. On the other hand, for each $x \in \Delta_v^m(c_0)$ there exists one and only one $y = (y_k) \in c_0$ such that

$$\begin{aligned}&x_n = v_n^{-1} \sum_{k=1}^{n} \binom{n+m-k-1}{m-1} y_k = v_n^{-1} \sum_{k=0}^{n} \binom{n+m-k-1}{m-1} y_k = 0, \\ &y_0 = 0,\end{aligned}$$

by (1) and Remark given above. Hence

$$\lim_n a_n x_n = \lim_n \sum_{k=0}^{n} \binom{n+m-k-1}{m-1} v_n^{-1} a_n y_k = 0, \quad \forall y \in c_0.$$

If we take

$$a_{nk} = \begin{cases} \binom{n+m-k-1}{m-1} v_n^{-1} a_n, & 1 \leqslant k \leqslant n, \\ 0, & k > n, \end{cases}$$

then we get

$$\lim_n \sum_{k=0}^{\infty} a_{nk} y_k = \lim_n \sum_{k=0}^{n} \binom{n+m-k-1}{m-1} v_n^{-1} a_n y_k = 0, \quad \forall y \in c_0.$$

Hence $A \in (c_0, c_0)$ and so $\sup_n \sum_{k=0}^n |a_{nk}| = \sup_n \sum_{k=0}^n \binom{n+m-k-1}{m-1} |v_n^{-1}||a_n| < \infty$. This completes the proof. □

Throughout the following theorems, we shall write b_k instead of $\sum_{j=k+1}^{\infty} v_j^{-1} a_j$.

Theorem 2.7. (a) *We put*

$$\begin{aligned}E_1(v) = \Bigg\{ a \in \omega \colon &\sum_{k=1}^{\infty} a_k v_k^{-1} \sum_{j=1}^{k-m} \binom{k-j-1}{m-1} \text{ is convergent}, \\ &\sum_{k=1}^{\infty} |b_k| \sum_{j=1}^{k-m+1} \binom{k-j-1}{m-2} < \infty \Bigg\}.\end{aligned}$$

Then $[D\Delta_v^m(\ell_\infty)]^\beta = E_1(v)$.

(b) *We put*

$$E_2(v) = \Biggl\{ a \in \omega\colon \sup_n \Biggl| \sum_{k=1}^{n} a_k v_k^{-1} \sum_{j=1}^{k-m} \binom{k-j-1}{m-1} \Biggr| < \infty,$$
$$\sum_{k=1}^{\infty} |b_k| \sum_{j=1}^{k-m+1} \binom{k-j-1}{m-2} < \infty \Biggr\}.$$

Then $[D\Delta_v^m(\ell_\infty)]^\gamma = E_2(v)$.

Proof. (a) If $x \in D\Delta_v^m(\ell_\infty)$ then there exists one and only one $y = (y_k) \in \ell_\infty$ such that

$$x_k = v_k^{-1} \sum_{j=1}^{k-m} \binom{k-j-1}{m-1} y_j, \qquad y_{1-m} = y_{2-m} = \cdots = y_0 = 0$$

for sufficiently large k, for instance $k > 2m$ by (1). Let $a \in E_1(v)$, and suppose that $\binom{-1}{-1} = 1$ (in some literature it is assumed that $\binom{r}{k} = 0$ for $k < 0$). Then we may write

$$\begin{aligned}
\sum_{k=1}^{n} a_k x_k &= \sum_{k=1}^{n} a_k \Biggl(v_k^{-1} \sum_{j=1}^{k-m} (-1)^m \binom{k-j-1}{m-1} y_j \Biggr) \\
&= (-1)^m \sum_{k=1}^{n-m} b_{k+m-1} \sum_{j=1}^{k} \binom{k+m-j-2}{m-2} y_j \\
&\quad - b_n \sum_{j=1}^{n-m} (-1)^m \binom{n-j-1}{m-1} y_j. \qquad (5)
\end{aligned}$$

Since $\sum_{k=1}^{\infty} |b_{k+m-1}| \sum_{j=1}^{k} \binom{k+m-j-2}{m-2} < \infty$, the series $\sum_{k=1}^{\infty} b_{k+m-1} \sum_{j=1}^{k} \binom{k+m-j-2}{m-2} y_j$ is absolutely convergent. Moreover we have $b_n \sum_{j=1}^{n-m} \binom{n-j-1}{m-1} \to 0$ as $n \to \infty$ by Lemma 2.1(ii), hence $\sum_{k=1}^{\infty} a_k x_k$ is convergent for all $x \in D\Delta_v^m(\ell_\infty)$, so $a \in [D\Delta_v^m(\ell_\infty)]^\beta$.

Let $a \in [D\Delta_v^m(\ell_\infty)]^\beta$. Then $\sum_{k=1}^{\infty} a_k x_k$ is convergent for each $x \in D\Delta_v^m(\ell_\infty)$. For the sequence $x = (x_k)$ defined by

$$x_k = \begin{cases} 0, & k \leqslant m, \\ v_k^{-1} \sum_{j=1}^{k-m} \binom{k-j-1}{m-1}, & k > m, \end{cases}$$

we may write

$$\sum_{k=1}^{\infty} a_k v_k^{-1} \sum_{j=1}^{k-m} \binom{k-j-1}{m-1} = \sum_{k=1}^{\infty} a_k x_k.$$

Thus the series $\sum_{k=1}^{\infty} a_k v_k^{-1} \sum_{j=1}^{k-m} \binom{k-j-1}{m-1}$ is convergent. This implies that $b_n \sum_{j=1}^{n-m} \binom{n-j-1}{m-1} \to 0$ as $n \to \infty$ by Lemma 2.1(ii).

Now let $a \in [D\Delta_v^m(\ell_\infty)]^\beta - E_1(v)$. Then $\sum_{k=1}^\infty |b_k| \sum_{j=1}^{k-m+1} \binom{k-j-1}{m-2}$ is divergent, that is, $\sum_{k=1}^\infty |b_k| \sum_{j=1}^{k-m+1} \binom{k-j-1}{m-2} = \infty$. We define the sequence $x = (x_k)$ by

$$x_k = \begin{cases} 0, & k \leqslant m, \\ v_k^{-1} \sum_{i=1}^{k-1} \operatorname{sgn} b_i \sum_{j=1}^{i-m+1} \binom{i-j-1}{m-2}, & k > m, \end{cases}$$

where $a_k > 0$ for all k or $a_k < 0$ for all k. Since $|\Delta_v^m x_k| = 1$ for $k > m$, it is trivial that $x = (x_k) \in D\Delta_v^m(\ell_\infty)$. Then we may write for $n > m$,

$$\sum_{k=1}^n a_k x_k = -\sum_{k=1}^m b_{k-1} \Delta_v x_{k-1} - \sum_{k=1}^{n-m} b_{k+m-1} \Delta_v x_{k+m-1} - b_n x_n v_n. \tag{6}$$

Since $(b_n x_n v_n) \in c_0$, from (6) now letting $n \to \infty$ we get

$$\begin{aligned} \sum_{k=1}^\infty a_k x_k &= \sum_{k=1}^\infty b_{k+m-1} \Delta_v x_{k+m-1} \\ &= \sum_{k=1}^\infty |b_{k+m-1}| \sum_{j=1}^k \binom{k+m-j-2}{m-2} = \infty. \end{aligned}$$

This contradicts to $a \in [D\Delta_v^m(\ell_\infty)]^\beta$. Hence $a \in E_1(v)$.

(b) can be proved by the same way as above, using Lemma 2.1(i). □

Lemma 2.8. $[D\Delta_v^m(\ell_\infty)]^\eta = [D\Delta_v^m(c)]^\eta$ *for* $\eta = \beta$ *or* γ.

Proof is easy.

Theorem 2.9. *Let* c_0^+ *denote the set of all positive null sequences.*

(a) *We put*

$$E_3(v) = \left\{ a \in \omega\colon \sum_{k=1}^\infty a_k v_k^{-1} \sum_{j=1}^{k-m} \binom{k-j-1}{m-1} u_j \text{ converges and } \sum_{k=1}^\infty |b_k| \sum_{j=1}^{k-m+1} \binom{k-j-1}{m-2} u_j < \infty,\ \forall u \in c_0^+ \right\}.$$

Then $[D\Delta_v^m(c_0)]^\beta = E_3(v)$.

(b) *We put*

$$E_4(v) = \left\{ a \in \omega\colon \sup_n \left| \sum_{k=1}^n a_k v_k^{-1} \sum_{j=1}^{k-m} \binom{k-j-1}{m-1} u_j \right| < \infty,\ \sum_{k=1}^\infty |b_k| \sum_{j=1}^{k-m+1} \binom{k-j-1}{m-2} u_j < \infty,\ \forall u \in c_0^+ \right\}.$$

Then $[D\Delta_v^m(c_0)]^\gamma = E_4(v)$.

Proof. (a) and (b) can be proved by the same way as Theorem 2.4, using Lemma 2.1(i) and (ii). □

The proof of the following result is a routine work.

Lemma 2.10.

(i) $[\Delta_v^m(\ell_\infty)]^\eta = [D\Delta_v^m(\ell_\infty)]^\eta$,
(ii) $[\Delta_v^m(c)]^\eta = [D\Delta_v^m(c)]^\eta$,
(iii) $[\Delta_v^m(c_0)]^\eta = [D\Delta_v^m(c_0)]^\eta$

for $\eta = \beta$ *or* γ.

3. Matrix transformations

Given any infinite matrix $A = (a_{nk})_{n,k=1}^\infty$ of complex numbers and any sequence $x = (x_k)$, we write $A_n(x) = \sum_{k=1}^\infty a_{nk}x_k$ $(n = 1, 2, \ldots)$ and $Ax = (A_n(x))_{n=1}^\infty$, provided the series $\sum_{k=1}^\infty a_{nk}x_k$ are convergent for each $n \in \mathbf{N}$.

The proof of the following result is a routine work in view of Theorem 2.7(a).

Theorem 3.1. *Let* $G = \ell_\infty$ *or* c *and* $H = \ell_\infty$ *or* c. *Then* $A = (a_{nk}) \in (\Delta_v^m(G), H)$ *if and only if*

(i) $(a_{nj})_n \in H$ $(1 \leqslant j \leqslant m)$,
(ii) $\left(\sum_{k=1}^\infty a_{nk} v_{k-1} \sum_{j=1}^{k-m} \binom{k-j-1}{m-1} j^{-r}\right)_n \in H$,
(iii) $\left(b_{nk} \sum_{j=1}^{k-m+1} \binom{k-j-1}{m-2} j^{-r}\right) \in (G, H)$,

where $b_{nk} = \sum_{i=k+1}^\infty a_{ni} v_i^{-1}$.

Theorem 3.2. *Let* $G = \ell_\infty$ *or* c *and* $H = \ell_\infty$, c *or* c_0. *Then* $A = (a_{nk}) \in (G, \Delta_v^m(H))$ *if and only if*

(i) $\sum_{k=1}^\infty |a_{nk}| < \infty$ *for each* n,
(ii) $C \in (G, H)$,

where $C = (c_{nk}) = (\Delta^{m-1} v_n a_{nk} - \Delta^{m-1} v_{n+1} a_{n+1,k})$.

The proof is omitted.

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